Multi-sided implicit surfacing with I-patches

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Outline

1 Introduction
- Motivation
- Previous work

2 I-patch

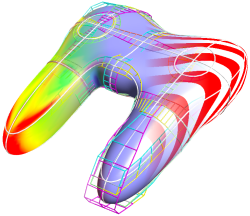
3 Constructing I-patches
- Boundary curves
- Primary & bounding surfaces
- Adjusting the interior

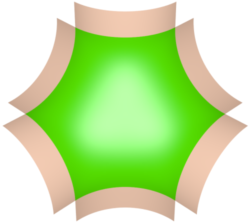
4 Applications

5 Conclusion

Implicit multi-sided patches

- Multi-sided surfaces used in many areas
  - Design
  - Hole filling & Vertex blending
- Exact representations →
  - explicit shape control
  - watertight connections
- Generally – n-sided parametric patches
- An interesting alternative: implicit patches
  - Easy inside-outside testing
  - Boolean operations
  - Efficient raytracing
  - Connect to regular implicit surfaces





Previous work

- **Bajaj and Ihm (1992)**
  - Fitting implicit surfaces based on boundary conditions
  - Handling point, curve and normal constraints
- **Li et al. (1990)**, **Hartmann (2001)**
  - Functional splines – multi-sided blends
  - Originally used for convex roundings
  - Later generalized for more complex blends
- Warren (1989) – Algebraic blends
- Rockwood (1989) – Displacement blending
- Gourmel et al. (2013) – Gradient-based blending

I-patch

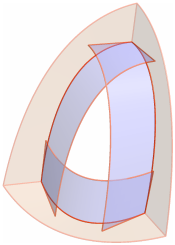
- Original publication:

T. Várady, P. Benkő, G. Kós, A. Rockwood
Implicit surfaces revisited–I-patches.
Geometric Modelling, Springer, Vienna, pp. 323–335, 2001.

- Current research: analyze basic scheme, distance-based formula, consistent ribbon orientation, special constructions, handling special cases

Basic formula

- $\{P_i(x,y,z)=0\}$ *implicit* primary surfaces,
  $\{B_i(x,y,z)=0\}$ *implicit* bounding surfaces
- $P_i \cap B_i \Rightarrow i^{\text{th}}$ boundary
- $I = \sum_{i=1}^{n} \left(w_i P_i \prod_{j \neq i} B_j^{k+1} \right) + w \prod_{i=1}^{n} B_i^{k+1}$
- Patch: $\{I(x,y,z)=0\}$
- k: degree of continuity
- Free parameters: $w_i, w \in \mathbb{R}$

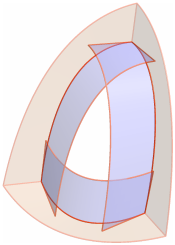


Example

3-sided G^1 I-patch:

$$w_1 P_1 B_2^2 B_3^2 + w_2 P_2 B_1^2 B_3^2 + w_3 P_3 B_1^2 B_2^2 + w B_1^2 B_2^2 B_3^2 = 0$$

- On boundary #i, $P_i(x,y,z) = 0$ and $B_i(x,y,z) = 0 \Rightarrow I(x,y,z) = 0$
- On boundary #i, $I = P_i G + B_i^2 H$ $(G, H : \mathbb{R}^3 \to \mathbb{R}) \Rightarrow \nabla I = const \cdot \nabla P_i$



Additional properties

$$I = \sum_{i=1}^{n} \left(w_i P_i \prod_{j \neq i} B_j^{k+1} \right) + w \prod_{i=1}^{n} B_i^{k+1}, \; w_i, w \in \mathbb{R}$$

- Interior C^∞
- G^k continuity to all P_i-s along the border
  - B_i-s derivatives do not affect $I^{(m)}, m \leq k$ on the boundary because of $k+1$ exponent
- Coincident bounding surfaces: slightly modified equation

Distance-based formulation

- Alternative "rational" form of I-patches

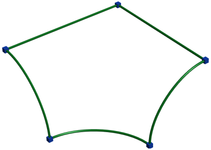
$$I = \sum_{i=1}^{n} \frac{w_i P_i}{B_i^{k+1}} + w$$

- a sum of weighted algebraic distances
- Unstable in the vicinity of the boundaries
  - near the i^{th} border: $I = P_i + \sum_{j \neq i} \left(\frac{w_j P_j}{B_j^{k+1}} + w \right) \cdot \frac{B_i^{k+1}}{w_i}$
- Useful for proving properties and setting the ribbon weights
  - I-patch touching three orthogonal elliptic cylinders reproduces an ellipsoid (see paper)

Constructing I-patches

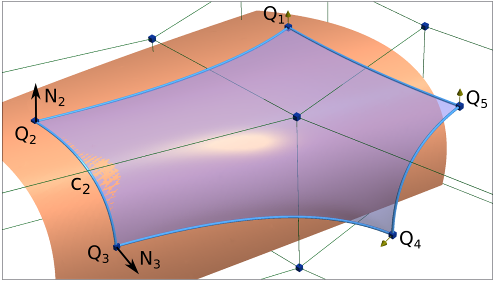
Boundary curves

- Input: corner points (Q_i) and tangent planes (implicit function: π_i)
- Boundary curves: smoothly connect corners
- Curves with implicit representation needed
  - If feasible, conics
  - Otherwise I-segments (2D implicit curves)

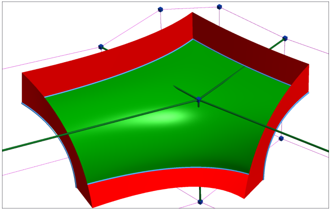


Implicit ribbons (primaries)

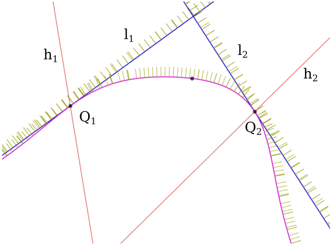
- Interpolate boundary curves (here $Q_2 - Q_3$)
- Consistent orientation with corner normals
- Common ribbon $\to G^1$ continuity between adjacent patches

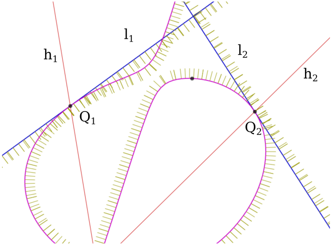


Orienting ribbons



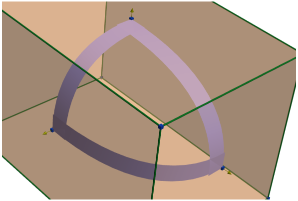
- Correct patch – consistent normal fence
- Positive half-spaces of the ribbons oriented accordingly
- 2D examples:



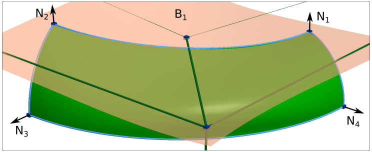


Implicit ribbons (primaries)

- Basic idea: Liming ribbon
- Quadratic equation combining the tangential planes and a cutting plane
  $(1-\lambda_i)\pi_i\pi_{i+1} - \lambda_i\tilde{\pi}_i^2$

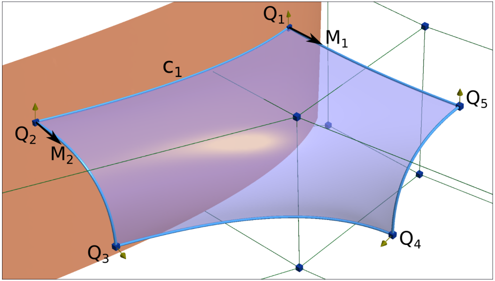


- I-segment boundary $\rightarrow$ I-patch ribbon
  $w_{i,1}\pi_i\mu_{i+1}^2 + w_{i,2}\pi_{i+1}\mu_i^2 + w_{i,0}\mu_i^2\mu_{i+1}^2$
  (μ-s: local bounding planes)

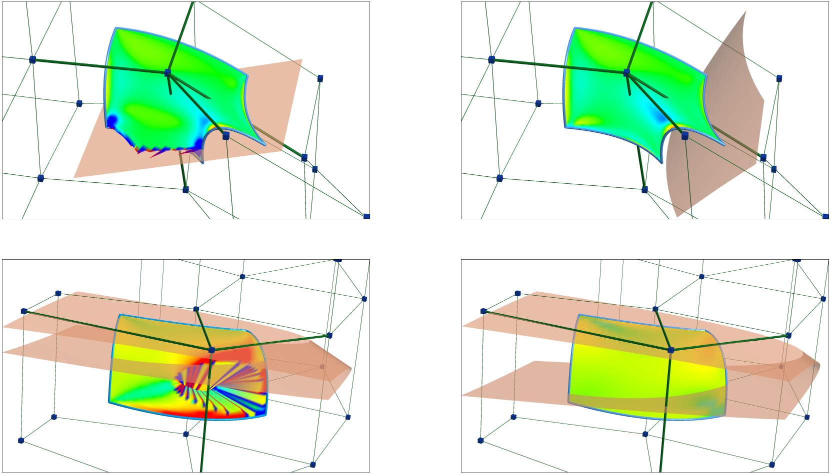


Implicit bounding surfaces

- Interpolate the boundary curve (here $Q_1 - Q_2$)
- Determines a half-space, where the I-patch is located
- Should not span a small angle with its ribbon

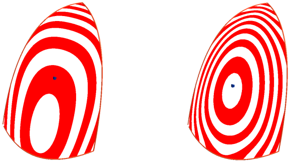


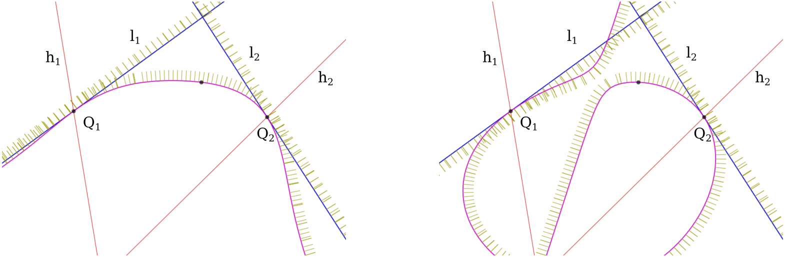
Detect & fix incorrect components



Scalar weights

- Weights (w_i) to adjust "fullness"
- E.g.: point approximation, fairing
  - Sign must not change (flips normal)





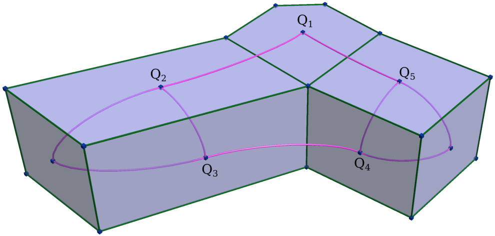
Default weights

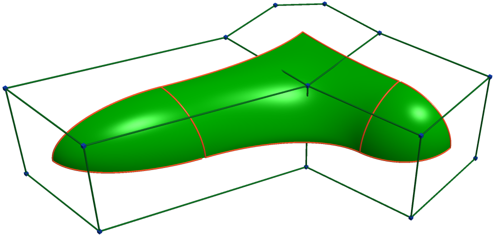
- Our heuristics: pick a reference point ($\mathbf{R}$) in the middle and set each weighted algebraic distance equal
  $d_i = \dfrac{w_i P_i(\mathbf{R})}{B_i^2(\mathbf{R})}$, $d_i = d_j, \quad (i, j = 1, \ldots, n)$
- Works well with good quality algebraic distance fields
- When needed – manual setting or automatic smoothing

Applications

Polyhedral design

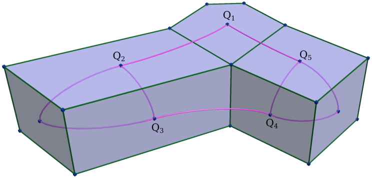
- Input: a control polyhedron
- Output: a smoothly connected composite surface of I-patches

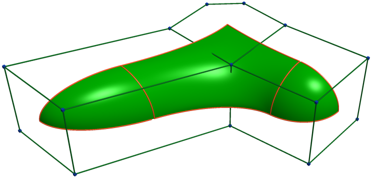




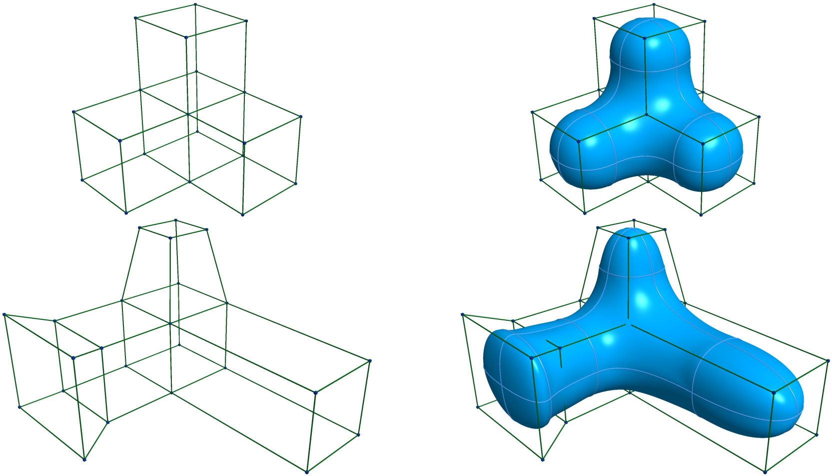
Polyhedral design

- Curve network
  - smoothly connects the centroids of the faces
  - a dual structure
- Ribbons
  - created exclusively from the polyhedron
  - G^1 connection automatically guaranteed

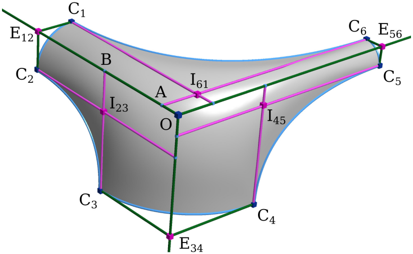




Polyhedral design



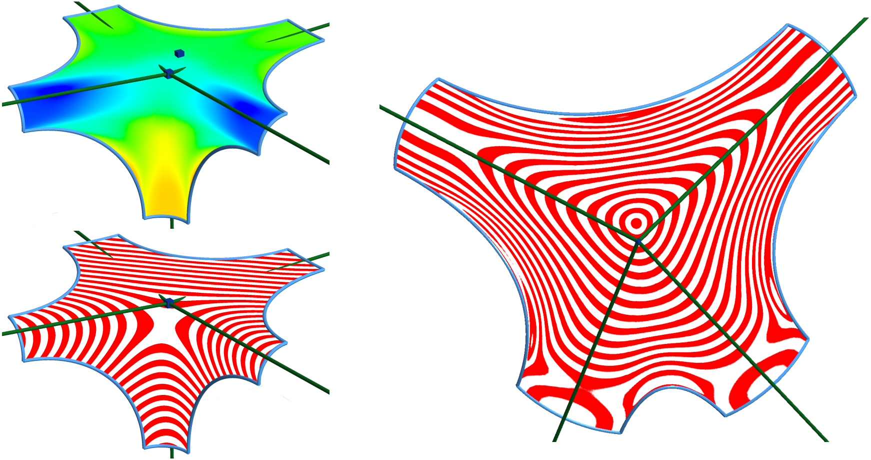
Setback vertex blending



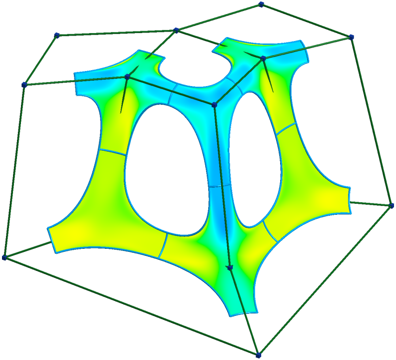
- Input: a vertex with blended edges defined by radii and setbacks
- Output: a multi-sided I-patch, connecting edge blends and primary faces

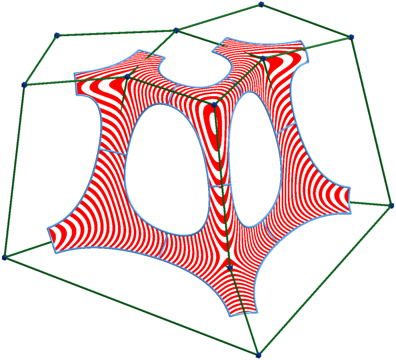
- Boundary loop – an alternating sequence of
  - profile curves (terminate blends)
  - and spring curves (attached to primary faces)

Setback vertex blending



Setback vertex blending





Conclusion

Summary

Building surfaces from I-patches

- Creating control data
- Creating primary and bounding surfaces
- Adjusting the surfaces

Applications

- Polyhedral design
- Setback vertex blends

Questions?

Thank you for your attention.